Analyzing Electric Fields Inside Cavities of Charged Spheres

Physics

The superposition principle is a fundamental concept in physics. It tells us that when a complex excitation can be broken down into smaller parts, the total response is simply the sum of the responses to each individual part. To better understand this principle, let's consider an electrostatic example.
Imagine a sphere with a radius of 1 meter that contains an empty spherical cavity with a radius of 0.25 meters. Inside this sphere, there is a positive volume charge density $\rho = 10^{-6} \, \frac{C}{m^3}$ The center of the cavity is at the distance $d = 0.5 \, \text{m}$ from the center of the charged sphere (Figure 1).

Figure 1 *- Positively charged sphere with an off-centered cavity*

According to the superposition principle, total field inside the cavity can be found by adding up individual fields of:

- A positively charged ($\rho = 10^{-6} \, \frac{C}{m^3}$), thoroughly filled sphere with a radius $a = 1 \, \text{m}$.
- A negatively charged ($\rho = -10^{-6} \, \frac{C}{m^3}$) sphere whose size and position match the cavity (Fig. 2).

Figure 2 *- Representation of an empty volume by a superposition of two opposite charge density domains*

The electric field within the interior of an isolated, uniformly charged sphere at a distance r from its center can be determined using Gauss' Law.

$$ E 4 \pi r^2 = \frac{1}{\epsilon_0} \frac{4}{3} \pi r^3 \rho $$

Which yields for a positive sphere:

$$ \stackrel{\rightarrow}{E} = \frac{\rho}{3 \epsilon_0} \stackrel{\rightarrow}{r_1} $$

And for a negative sphere:

$$ \stackrel{\rightarrow}{E}_{-} = - \frac{\rho}{3 \epsilon_0} \stackrel{\rightarrow}{r_2} $$

Where vectors $\stackrel{\rightarrow}{r_1}$ and $\stackrel{\rightarrow}{r_2}$ are as defined in Figure 3.

Figure 3 *- Relationship between the individual Electric field directions and the vector representing the cavity offset*

Therefore, the total electric field in the cavity can be computed as:

$$ \stackrel{\rightarrow}{E} = \stackrel{\rightarrow}{E}_{+} + \stackrel{\rightarrow}{E}_{-} = \frac{\rho}{3 \epsilon_0} \left(\stackrel{\rightarrow}{r_1} - \stackrel{\rightarrow}{r_2} \right) $$

$$ \stackrel{\rightarrow}{E} = \frac{\rho}{3 \epsilon_0} \stackrel{\rightarrow}{d} $$

From the last equation, it can be concluded that the electric field in the cavity is constant with a direction $\stackrel{\rightarrow}{d}$ and that its magnitude (for $d = 0.5 \, \text{m}$ and $\rho = 10^{-6} \, \text{C/m}^3$) is $E = 18.832 \, \frac{\text{kV}}{\text{m}}$ The field magnitude depends only on the value of the charge density and the distance by which the center of the cavity is offset from the center of the sphere.

Model

Creating an Air domain

In EMS, electromagnetic analysis often necessitates the modeling of the surrounding air regions because the electromagnetic field can extend beyond the parts of the simulated system. To incorporate the air domain into the assembly, you should follow these steps:

1. Select the **Air** part in the Solidworks feature manager
2. Click Edit component in the Solidworks Assembly tab
3. In Solidworks menu click Insert/Molds/Cavity
4. In the cavity feature manager, select **Charged sphere** and **Cavity** as the Design Components
5. Click OK .

Figure 4 *- 3D model of sphere with a spherical cavity together with surrounding air domain*

The simulation is performed as the EMS **Electrostatic study** . Air is used as a material for all parts.
(To see how to assign materials, see the “Computing capacitance of a multi-material capacitor” example).

Boundary conditions

To simulate the electric field, you should follow these boundary condition assignments:

1. Assign a Charge Density boundary condition to the large sphere. This will specify the charge distribution on the surface of the sphere.

2. Assign a Fixed Voltage boundary condition to the face of the Air region. This will set a fixed voltage on the specified face of the air region, which will interact with the charge distribution on the large sphere to determine the electric field.

To assign a charge density to the **Charged sphere**:

1. In the EMS manger tree, Right-click on the **Load/Restraint** , select **Charge density** , then choose **Volume**.
2. Click inside **the Bodies Selection** box and then select the **Charged sphere**.
3. In the **Charge Density** tab, type **1e-006**.
4. Click OK .

To see how to assign 0 Volt to the face of the Air region, see “Force in a capacitor” example.

Results

To display the variation of the electric field along the axis that connects the center of the Charged sphere and the center of the cavity:

1. In the EMS manger tree, Under **Results** , right click on the **Electric Field** folder and select **2D Plot** then choose **Linear.**
2. The **2D Electric Field** Property Manager Page appears.
3. In the **Select points** tab, select the start and the end points.
4. In the **Number of Points** tab, type 1000.
5. Click OK .

In the obtained curve (Figure 5), it’s clear that the electric field in the cavity is constant, and its value is $E = 18.797 \, \text{K volts}$, which closely matches the theoretical result. The field in Figure 5 steadily increases with the radius until it meets the cavity $r = 250 \, \text{mm}$ and then remains unchanged through the cavity (till $r = 750 \, \text{mm}$). The field peaks at the surface of the sphere ($r = 1000 \, \text{mm}$) and then it drops with the square of radius.

Figure 5 *- Electric field vs radius*

Conclusion

This application note delves into the superposition principle in physics through the lens of an electrostatic example, involving a positively charged sphere with an off-centered cavity. By applying the superposition principle, the study breaks down the complex excitation into two simpler scenarios: a fully charged sphere and a negatively charged sphere matching the cavity's dimensions and position. Utilizing Gauss' Law, the electric fields of both positive and negative spheres are calculated, leading to a determination of the total electric field inside the cavity.
The EMS software plays a pivotal role in this analysis, facilitating the simulation of the electric field within the sphere's cavity. The modeling process includes the creation of an air domain to account for electromagnetic field extensions beyond the physical components. Essential steps such as applying material properties, defining boundary conditions like charge density and fixed voltage, and executing an Electrostatic study are outlined. Results from the simulation confirm the theoretical predictions, showcasing a constant electric field within the cavity, with its magnitude directly influenced by the charge density and the cavity's offset from the sphere's center.

In conclusion, this application note illustrates the power of EMS in validating theoretical concepts like the superposition principle and Gauss' Law in electrostatics, providing a comprehensive understanding of electric fields within charged bodies. This convergence of theoretical physics with simulation tools not only enhances our grasp of fundamental principles but also demonstrates the utility of software like EMS in practical, complex problem-solving within electromagnetic studies.

References

[1] http://jkwiens.com/2007/10/24/answer-electric-field-of-a-nonconducting-sphere-with-a-spherical-cavity/